Tanjak: Journal of Education and Teaching
Volume 7, Issue 2, August 2026
DOI: 10.35961/jg.v7i2.3074

p-ISSN: 2716-4098
e-ISSN: 2720-8966

Identifying the Factor Structure of Students' Algebraic Thinking Through an Area Model Supported by PhET Interactive Simulation

Kaila Tazkia[1], Shinta Yuhanida[2], Muhammad Ihsan Idzharuddien[3], Rina Oktaviyanthi[4]*, Ria Noviana Agus[5], Giyanti[6]

[1] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: kailatazzkia@gmail.com
[2] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: shintayhnd@gmail.com
[3] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: idzharuddieni@gmail.com
[4] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: rinaokta@unsera.ac.id
[5] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: riaagus@unsera.ac.id
[6] Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Serang Raya, Serang, Banten, Indonesia.
Email: giyanti@unsera.ac.id

***Corresponding Author**
Email: rinaokta@unsera.ac.id

Article History
Received : 14 July 2026
Revised : 04 August 2026
Accepted : 09 August 2026
Published : 27 August 2026

Abstract
This study aims to identify the factor structure of students' algebraic thinking skills within an area model learning context supported by PhET Interactive Simulation. A quantitative approach using exploratory factor analysis was applied to data from 128 students based on 16 indicators representing four aspects of algebraic thinking: generalization, representation, transformation, and conceptual meaning. The analysis involved the Kaiser-Meyer-Olkin (KMO) test, Bartlett's test of sphericity, measures of sampling adequacy (MSA), principal component analysis, eigenvalue criteria, and Varimax rotation. The initial analysis produced a KMO value of .631, while four variables (G1, G3, G4, and R1) were removed because their MSA values were below .50. The analysis of the remaining 12 variables produced a KMO value of .689 and a significant Bartlett's test ($p < .001$). Four components with eigenvalues greater than 1 were retained and accounted for 62.06% of the total variance before rotation. The rotated solution revealed three interpretable dimensions: a visual-representational factor integrating generalization and

representation, a procedural-transformational factor, and a conceptual-meaning factor. The fourth component had no clearly dominant group of indicators and was therefore interpreted cautiously as a residual component. These findings indicate that students' algebraic thinking in the examined instructional context is multidimensional, with visual-representational, procedural-transformational, and conceptual dimensions showing distinct patterns.

Keywords: Algebraic thinking; area model; exploratory factor analysis; PhET interactive simulation; visual-interactive learning.

Introduction

Algebraic thinking is a fundamental mathematical competence, yet its abstract nature continues to pose substantial challenges for students (Kriegler, 1999; Kieran, 2022; Sun et al., 2023). Previous studies have shown that visual approaches can help address difficulties in learning algebra because students with stronger mathematical visual abilities are better able to transform information across multiple representations (Vale & Barbosa, 2023; Hayes & Proulx, 2024). This process not only supports problem solving but also deepens conceptual understanding through visual stimulation that influences students' cognitive processing (Lowrie & Logan, 2023). In this context, the area model functions not merely as an illustration but as an accurate representation of algebraic relationships and structures (Ünal et al., 2023; Wilkie, 2024). Such an approach has the potential to increase motivation, encourage the exploration of mathematical ideas, reduce cognitive load, and support higher-order thinking. Strengthening algebraic thinking through visual representations is therefore important in mathematics education.

Despite this potential, students continue to experience difficulties in algebraic thinking, particularly when connecting visual and symbolic representations. They often struggle to recognize patterns and formulate generalizations (Raviv et al., 2022), for example, to understand that $x(x+3)$ can be represented as an area and expanded into x^2+3x. In terms of representation, students may be unable to translate visual models into symbolic forms or vice versa (Prain & Tytler, 2022; Wahab, 2026), such as relating $(x+2)(x+3)$ to x^2+5x+6. Difficulties in transformation arise when students are required to manipulate algebraic expressions flexibly (Rafiepour et al., 2023; Harel, 2025), including simplifying and factoring quadratic expressions (Kabar, 2023; Ngoveni, 2025). In relation to conceptual meaning, students may also struggle to connect algebraic forms with their mathematical meanings, for example, by interpreting the coefficients and constant in ax^2+bx+c as quantitative relationships (Pelayo et al., 2023; Artasari & Oktaviyanthi, 2024). These difficulties may hinder the development of algebraic thinking and the understanding of more advanced mathematical concepts.

One approach to addressing these difficulties is the use of visual-interactive learning media, such as an area model supported by PhET Interactive Simulation (Perkins, 2020; Laurence, 2022; Artasari et al., 2024). This medium enables algebraic relationships to be visualized concretely and dynamically (de Sousa & Alves, 2022; Akinoso et al., 2025), while simultaneously supporting generalization, representation, and transformation (Yao, 2022; Alam & Mohanty, 2024). The area model also supports conceptual meaning by connecting length, width, and area with variables and multiplication operations (Álvaréz et al., 2024; Lee & Yeo, 2026). In addition, the interactive features of PhET allow students to manipulate variables directly and observe changes in algebraic forms dynamically. These features make it a potentially useful medium for supporting the development of algebraic thinking.

However, the implementation of visual-interactive media has not fully revealed the structure of students' algebraic thinking, including its dominant and less developed aspects and the relationships among them. An analytical approach is therefore required to identify patterns of association and the latent structure underlying the observed variables. Exploratory Factor Analysis (EFA) is relevant for identifying underlying dimensions based on observed indicators (Osborne & Kellow, 2008; Sürücü et al., 2022; Nájera et al., 2025). EFA can reduce a set of variables into a smaller number of more representative factors (Sürücü et al., 2022; Brandenburg & Papenberg, 2024) and group highly correlated indicators into latent constructs, thereby revealing relationships among different aspects of the measured competence (Osborne & Kellow, 2008; Griffith, 2024).

Given the need to support visually based algebraic thinking and the limited information available about its underlying factor structure, this study aims to identify the factor structure of students' algebraic thinking through the use of an area model supported by PhET Interactive Simulation. The findings are expected to provide a comprehensive description of the dimensions of algebraic thinking demonstrated by students in this learning context.

Metode

To identify the factor structure of students' algebraic thinking, this study employed a quantitative approach using exploratory factor analysis (Sürücü et al., 2022; Rogers, 2022). Data were obtained from students' responses to an area model-based worksheet supported by PhET Interactive Simulation. The score for each item reflected the accuracy and completeness of the response and served as an indicator of students' algebraic thinking. The study involved 128 students who had participated in instruction using the learning medium. The observed variables represented four dimensions of algebraic thinking: generalization, representation, transformation, and conceptual meaning (see Table 1).

Table 1. Indicators of Algebraic Thinking

Sub-Variable	Task Description in the Worksheet
Generalization (G) Ability to identify patterns and formulate general expressions	
G1	Students were given a rectangular area model consisting of the sections $x \times x$, $x \times 3$, dan $2 \times x$. They were asked to determine the algebraic expression represented by the model and explain the resulting pattern.
G2	Students examined area model patterns for $(x+1)(x+2)$, $(x+2)(x+3)$, and $(x+3)(x+4)$. They used PhET to visualize the patterns and determine their general form.
G3	Students were given an area model with length $(x+a)$ and width $(x+b)$. They were asked to determine the relationship between the variables and express it as a general algebraic expression.
G4	Students were given three different area models representing expressions of the form $(x+p)(x+q)$. They were asked to determine the general form and explain how the pattern could be generalized.
Representation (R) Ability to express concepts in multiple forms	
R1	Students were given an area model divided into several rectangles. They were asked to write the corresponding algebraic expression.
R2	Students used PhET to represent the expression $(x+3)(x+2)$ as an area model and explained the relationship between the visual model and its symbolic form.
R3	Students were given the expression x^2+5x+6. They were asked to represent it as an area model and explain the relationship between its symbolic and visual forms.
R4	Students were given an area model and asked to determine the corresponding algebraic equation and explain how each section of the area represented a component of the expression.
Transformation (T) Ability to manipulate algebraic expressions	
T1	Students used an area model to expand $(x+2)(x+4)$. They were asked to show the expansion process and state the final result.
T2	Students were given the expression $x^2+7x+10$. They used an area model to factor the expression.
T3	Students were given an area model representing x^2+6x+8. They were asked to simplify the algebraic expression and explain the process.

Sub-Variable	Task Description in the Worksheet
T4	Students converted $(x + 1)(x + 5)$ into standard form using an area model and explained the equivalence between the two forms.
Conceptual Meaning (C) Ability to connect representations with mathematical meaning	
C1	Students explained the meaning of each section in an area model representing $(x + 3)(x + 2)$.
C2	Students explained how an area model could be used to understand algebraic multiplication.
C3	In the context of the area of a rectangular garden with dimensions $(x+2)$ and $(x+3)$, students explained the meaning of the resulting algebraic expression.
C4	Students used PhET to change the constant values in $(x + a)(x + b)$ and explained how these changes affected the form and structure of the algebraic expression.

The factor analysis procedure followed Hair et al. (2020) and included the Kaiser-Meyer-Olkin test, Bartlett's test of sphericity, measures of sampling adequacy, factor extraction, determination of the number of factors, rotation, and interpretation. The analyses were conducted using SPSS and supported by Python-based computation. The analytical procedure is presented in Figure 1.

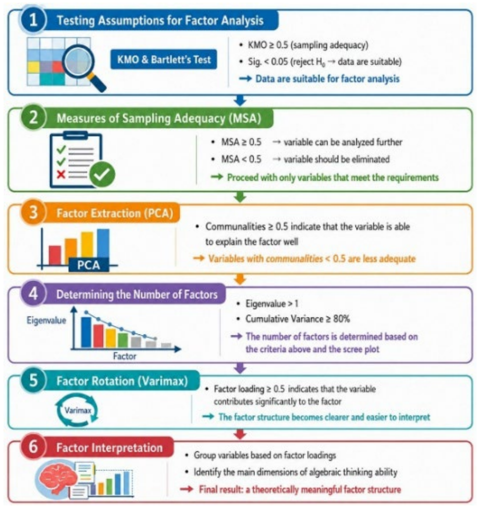

Figure 1. Factor Analysis Framework (the visualization was developed with AI assistance)

The research instrument was an area model-based worksheet supported by PhET Interactive Simulation, designed to assess generalization, representation, transformation, and conceptual meaning (see Figure 2).

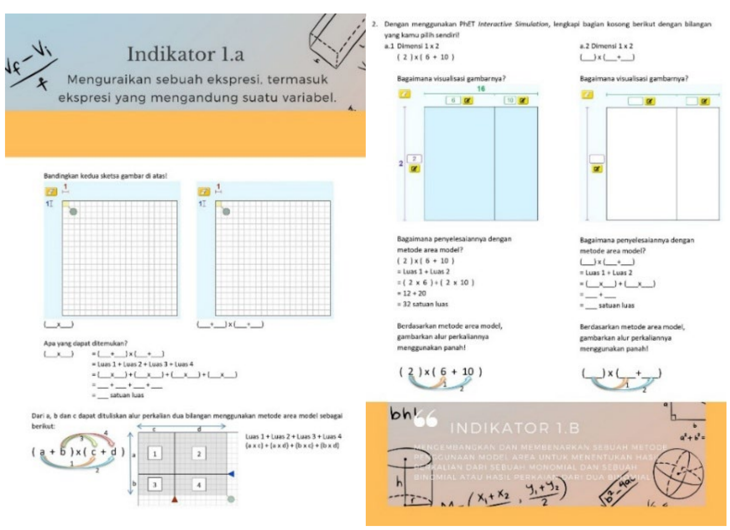

Figure 2. Sample Area Model Worksheet

Results and Discussion

Assessment of Factor Analysis Assumptions

The initial output of the exploratory factor analysis was used to assess the suitability of the variables for factor analysis. As shown in Table 2, the KMO value was 0.631, which falls within the mediocre range of 0.6–0.7. This result indicates that the data remained acceptable for factor analysis, although further evaluation of particular variables was required.

Table 2. KMO and Bartlett's Test for 16 Variables

KMO and Bartlett's Test		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.631
Bartlett's Test of Sphericity	Approx. Chi-Square	484.019
	df	120
	Sig.	.000

The significance value of Bartlett's test was .000, which was lower than .05, indicating that H_0 was rejected and that the correlation matrix was not an identity matrix. This result shows that significant relationships existed among the variables, allowing the factor analysis to proceed. Thus, the variables examined in this study demonstrated sufficient intercorrelations for the identification of a factor structure.

Measures of Sampling Adequacy (MSA)

The second procedure involved examining the Measures of Sampling Adequacy (MSA) to assess the suitability of each variable for factor analysis in terms of both sample adequacy and the strength of its correlations with the other variables. This test was also used to identify variables with weak correlations that needed to be eliminated to produce a more appropriate factor solution. The MSA values were obtained from the Anti-image Matrices in the SPSS output, as presented in Table 3.

Table 3. MSA Values for 16 Variables

Anti-image Corr	G1	G2	G3	G4	R1	R2	R3	R4	T1	T2	T3	T4	C1	C2	C3	C4
G1	.381[a]	.214	-.102	-.088	.145	.062	.071	-.055	-.132	.041	.063	.092	-.144	.058	.077	-.031
G2	.214	.602[a]	-.176	.118	.202	-.041	-.033	-.064	.089	.133	.154	-.098	.122	.084	.091	-.112
G3	-.102	-.176	.413[a]	-.221	.038	-.187	-.144	-.301	-.089	-.112	-.118	.095	-.072	-.061	-.084	.104
G4	-.088	.118	-.221	.490[a]	-.155	.201	.073	.144	.102	.244	-.063	-.221	.081	.173	-.052	-.203
R1	.145	.202	.038	-.155	.465[a]	.153	.044	-.121	-.088	.201	.231	-.074	.105	.184	.210	-.091
R2	.062	-.041	-.187	.201	.153	.669[a]	.142	-.066	.034	.151	-.044	-.022	.072	.143	-.061	-.018
R3	.071	-.033	-.144	.073	.044	.142	.746[a]	-.233	.018	.151	.202	-.071	.014	.138	.177	-.065
R4	-.055	-.064	-.301	.144	-.121	-.066	-.233	.616[a]	.028	-.101	-.109	.066	.039	-.095	-.104	.071
T1	-.132	.089	-.089	.102	-.088	.034	.018	.028	.699[a]	.041	-.052	-.121	-.101	.061	-.048	-.098
T2	.041	.133	-.112	.244	.201	.151	.151	-.101	.041	.540[a]	.066	-.084	.057	.152	-.089	-.062
T3	.063	.154	-.118	-.063	.231	-.044	.202	-.109	-.052	.066	.691[a]	-.221	-.063	.071	.198	-.211
T4	.092	-.098	.095	-.221	-.074	-.022	-.071	.066	-.121	-.084	-.221	.707[a]	.133	-.092	-.071	.058
C1	-.144	.122	-.072	.081	.105	.072	.014	.039	-.101	.057	-.063	.133	.709[a]	.221	-.034	-.102
C2	.058	.084	-.061	.173	.184	.143	.138	-.095	.061	.152	.071	-.092	.221	.564[a]	.081	-.066
C3	.077	.091	-.084	-.052	.210	-.061	.177	-.104	-.048	-.089	.198	-.071	-.034	.081	.669[a]	-.188
C4	-.031	-.112	.104	-.203	-.091	-.018	-.065	.071	-.098	-.062	-.211	.058	-.102	-.066	-.188	.510[a]

Table 3 shows that, of the 16 variables analyzed, four variables had MSA values below .50, namely G1, G3, G4, and R1. Based on the MSA criteria presented in Figure 1, these four variables had weak intercorrelations, indicating that they could not be adequately predicted by the other variables and were therefore unsuitable for inclusion in the subsequent factor analysis. Accordingly, G1, G3, G4, and R1 were removed from the model, and the remaining 12 variables were reanalyzed.

The results of the reanalysis indicated an increase in the KMO value, as presented in Table 4. The removal of variables that did not meet the required criteria increased the KMO value from the mediocre category toward a more acceptable level. This result suggests that the 12 retained variables were more suitable for factor analysis.

Table 4. KMO and Bartlett's Test for 12 Variables

KMO and Bartlett's Test		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.689
Bartlett's Test of Sphericity	Approx. Chi-Square	379.266
	df	66
	Sig.	.000

The MSA values for the 12 reanalyzed variables were all above .50, as shown in Table 5. This result indicates that all remaining variables could be sufficiently predicted by the other variables and were suitable for further factor analysis. The preceding variable-reduction procedure therefore improved the adequacy of the relationships among the variables and produced a more representative analytical model.

Table 5. MSA Values for 12 Variables

Anti-image Corr	G2	R2	R3	R4	T1	T2	T3	T4	C1	C2	C3	C4
G2	**.516**[a]	0.027	0.011	-0.080	0.120	-0.072	-0.126	-0.096	0.033	0.061	-0.045	0.052
R2	0.027	**.617**[a]	-0.034	0.194	-0.259	0.100	0.137	-0.134	-0.120	0.084	0.071	-0.063
R3	0.011	-0.034	**.580**[a]	0.012	-0.106	-0.277	-0.056	0.038	-0.075	0.091	0.102	-0.081
R4	-0.080	0.194	0.012	**.664**[a]	0.212	0.004	0.106	0.060	-0.073	-0.055	-0.088	0.049
T1	0.120	-0.259	-0.106	0.212	**.633**[a]	-0.117	-0.056	-0.100	0.131	0.072	-0.066	-0.041
T2	-0.072	0.100	-0.277	0.004	-0.117	**.507**[a]	0.154	-0.102	-0.214	0.093	0.081	-0.057
T3	-0.126	0.137	-0.056	0.106	-0.056	0.154	**.588**[a]	0.229	-0.027	0.065	0.144	-0.098
T4	-0.096	-0.134	0.038	0.060	-0.100	-0.102	0.229	**.610**[a]	0.105	-0.072	0.083	0.061
C1	0.033	-0.120	-0.075	-0.073	0.131	-0.214	-0.027	0.105	**.570**[a]	0.187	-0.034	0.098
C2	0.061	0.084	0.091	-0.055	0.072	0.093	0.065	-0.072	0.187	**.645**[a]	0.081	-0.066
C3	-0.045	0.071	0.102	-0.088	-0.066	0.081	0.144	0.083	-0.034	0.081	**.559**[a]	-0.102
C4	0.052	-0.063	-0.081	0.049	-0.041	-0.057	-0.098	0.061	0.098	-0.066	-0.102	**.602**[a]

Factor Extraction Using PCA

The purpose of factor extraction was to generate a set of factors capable of explaining the relationships among the variables. In this study, Principal Component Analysis (PCA) was used, with reference to the communality values, which indicate the extent to which each variable is accounted for by the extracted factors. Communality values of .50 or higher indicate that a variable is adequately represented by the factor solution.

Table 6. Communality Values

Communalities		
	Initial	**Extraction**
G2	1.000	.352
R2	1.000	.315
R3	1.000	.333
R4	1.000	.486
T1	1.000	.312
T2	1.000	.322
T3	1.000	.249
T4	1.000	.354
C1	1.000	.294
C2	1.000	.419
C3	1.000	.209

Extraction Method: Principal Component Analysis.

Based on Table 6, some variables had communality values above .50, whereas others had values below .50, indicating variation in the extent to which the variables were represented by the extracted factors. Nevertheless, all variables were retained because the overall model was considered capable of explaining the relationships among them. Higher communality values indicate that a variable is better represented by the factors, whereas lower values indicate less adequate representation. For example, G2 had a communality value of .352, indicating that 35.2% of its variance was explained by the extracted factors.

Determination of the Number of Factors

The number of factors was determined based on eigenvalues, which indicate the contribution of each component to the factor solution. These values are presented in the *Total Variance Explained* table (Table 7), which also reports the percentage of variance explained by each factor. Factors with eigenvalues greater than 1 were retained, whereas those with eigenvalues below 1 were excluded because of their limited contribution. As shown in Table 7, four factors had eigenvalues greater than 1 and were therefore retained because they accounted for a substantial proportion of the variance in the data.

Table 7. Variance Explained by the Components

Total Variance Explained									
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.222	26.850	26.850	3.222	26.850	26.850	2.890	24.083	24.083
2	1.791	14.926	41.776	1.791	14.926	41.776	2.615	21.792	45.875
3	1.417	11.809	53.585	1.417	11.809	53.585	2.232	18.600	64.475
4	1.017	8.472	62.057	1.017	8.472	62.057	2.126	17.717	82.192
5	0.942	7.850	69.907						
6	0.801	6.678	76.585						
7	0.733	6.110	82.695						
8	0.684	5.700	88.395						
9	0.612	5.100	93.495						
10	0.512	4.267	97.762						
11	0.168	1.398	99.160						
12	0.101	0.840	100.000						

Extraction Method: Principal Component Analysis.

As presented in Table 7, four factors had eigenvalues greater than 1: Factor 1 (3.222), Factor 2 (1.791), Factor 3 (1.417), and Factor 4 (1.017). The four factors accounted for 26.850% of the variance for Factor 1, 41.776% cumulatively for Factors 1 and 2, and 62.057% cumulatively before rotation. The cumulative percentage after rotation was reported as 82.192%. These results suggest that the extracted factors adequately represented the variance in the original variables. The scree plot in Figure 3 also supported the retention of four factors, as the curve began to level off after the fourth component, indicating that subsequent components contributed relatively little additional variance.

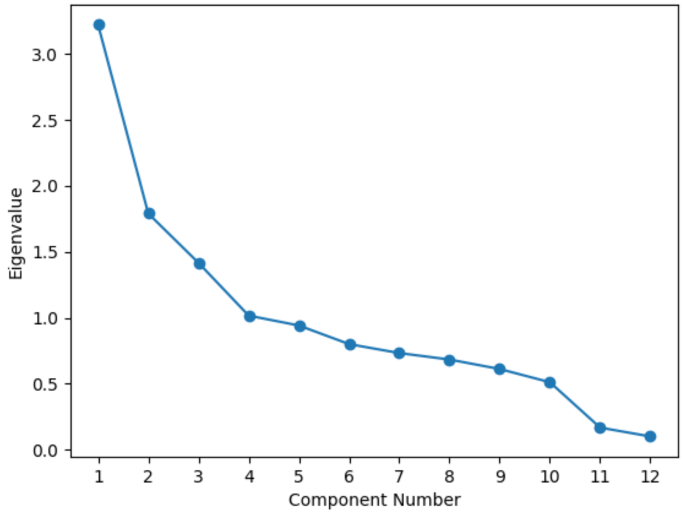

Figure 3. Scree Plot of the Extracted Components

Factor Rotation

The factor loadings indicate the strength of the relationship between each observed variable and the extracted factors. Table 8 presents the four-component solution and the corresponding component matrix. The values in the matrix represent the correlations between each variable and the four extracted components.

Table 8. Component Matrix

Component Matrix[a]				
	Component			
	1	2	3	4
G2	0.412	0.318	0.221	0.145
R2	0.365	0.402	0.198	0.122
R3	0.421	0.289	0.256	0.101
R4	0.488	0.201	0.233	0.144
T1	0.356	0.390	0.188	0.210
T2	0.378	0.345	0.201	0.187
T3	0.332	0.298	0.276	0.164
T4	0.401	0.311	0.255	0.132
C1	0.290	0.215	0.421	0.208
C2	0.345	0.188	0.398	0.221
C3	0.267	0.156	0.442	0.209
Extraction Method: Principal Component Analysis.				
a. 4 components extracted.				

As shown in Table 8, several variables had relatively high loadings on more than one factor, resulting in ambiguity in their classification. For example, T2 loaded at 0.378 on Factor 1 and 0.345 on Factor 2. Similar patterns were observed for T4 and R3. These cross-loadings indicated that the initial factor structure could not yet be interpreted clearly. Factor rotation was therefore required to obtain a simpler structure in which each variable had a dominant loading on one factor.

Varimax rotation was applied to maximize the variance of the loadings so that each variable would load strongly on one factor and weakly on the others. The rotated solution presented in Table 9 shows that each variable had a dominant loading on one factor, resulting in a clearer and more interpretable factor structure.

Table 9. Varimax-Rotated Component Matrix

Rotated Component Matrix[a]				
	Component			
	1	2	3	4
R3	**.745**	.144	.121	.088
R4	**.781**	.109	.098	.076
T4	**.703**	.215	.132	.090
R2	.182	**.734**	.144	.102
T1	.210	**.768**	.121	.115
T2	.198	**.721**	.133	.109
T3	.176	**.689**	.165	.121
C1	.132	.145	**.701**	.188
C2	.121	.133	**.728**	.201
C3	.109	.121	**.756**	.187
C4	.102	.115	**.712**	.214

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 5 iterations.

Table 9 shows that the dominant loading of each variable exceeded .50, indicating a strong association between the retained variables and their respective components. The rotated factor solution was therefore used as the basis for interpreting the factor structure in the subsequent stage.

Factor Interpretation

The factors derived from the preceding exploratory factor analysis procedures were interpreted as follows:

a. Factor 1 was strongly associated with G2 (generalization), R3 and R4 (representation), and T4 (transformation). It was therefore identified as a factor dominated by visual representation. This factor accounted for 26.85% of the variance, with the highest loading observed for R4 (.781).
b. Factor 2 was strongly associated with R2 (representation) and T1, T2, and T3 (transformation). It was therefore identified as a procedural-transformational factor representing the ability to manipulate algebraic expressions. This factor accounted for 14.93% of the variance, with the highest loading observed for T1 (.768).
c. Factor 3 was strongly associated with C1, C2, C3, and C4, which reflected students' ability to interpret the conceptual meaning of algebraic representations. This factor accounted for 11.81% of the variance, with the highest loading observed for C3 (.756).
d. Factor 4 did not contain a dominant group of variables and was therefore categorized as a weak or residual factor. Although it had an eigenvalue of 1.017 and accounted for 8.47% of the variance, its contribution to the main factor structure was relatively limited.

The component transformation matrix was subsequently examined, as presented in Table 10. High diagonal values of at least .80 were interpreted as indicating strong relationships between the original and rotated component axes. These values suggested that the rotated factors were sufficiently distinct and could be used for further interpretation.

Table 10. Component Transformation Matrix

Component Transformation Matrix				
Component	**1**	**2**	**3**	**4**
1	**.872**	.214	.398	-.102
2	-.295	**.918**	.126	.063
3	.336	.201	**.887**	-.174
4	.158	.074	-.142	**.951**

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

The exploratory factor analysis initially examined four aspects of algebraic thinking represented by 16 sub-variables. Following the removal of four variables, 12 variables were retained in the factor solution. The resulting factors were classified as dominant, moderate, adequate, and weak factors. Factor 1 was represented by G2, R3, R4, and T4, indicating an integration of generalization, visual representation, and algebraic transformation. It accounted for the largest proportion of variance, at 26.85%, and had its highest loading on R4 (.781). This pattern suggests that visual representations based on the area model were prominent within the factor structure identified from students' responses.

Factor 2 was represented by R2, T1, T2, and T3, reflecting procedural abilities related to algebraic manipulation, including expansion, factorization, and simplification. This factor accounted for 14.93% of the variance, with the highest loading on T1 (.768). Factor 3 was associated with C1–C4 and was identified as a conceptual-meaning factor, reflecting students' ability to interpret the meaning of algebraic representations. It accounted for 11.81% of the variance, with the highest loading on C3 (.756). Factor 4 did not contain a dominant group of variables and was therefore categorized as a weak or residual factor. Its contribution of 8.47% may indicate that some aspects of algebraic thinking were not represented as clearly as the visual, procedural, and conceptual dimensions.

The summary in Table 11 shows that generalization and representation were grouped within the dominant factor, transformation was primarily represented in the second factor, and conceptual meaning emerged as a relatively independent dimension. All retained variables had dominant loadings above .50.

Table 11. Summary of the Algebraic Thinking Factor Structure

Aspect	Sub-Variable	Factor	Factor Loading	Interpretation
Generalization	G2	1	.712	Significant
Representasi	R3	1	.745	Significant
	R4	1	.781	Significant
	R2	2	.734	Significant
Representation	T4	1	.703	Significant
	T1	2	.768	Significant
	T2	2	.721	Significant
	T3	2	.689	Significant
Conceptual Meaning	C1	3	.701	Significant
	C2	3	.728	Significant
	C3	3	.756	Significant
	C4	3	.712	Significant

Previous studies have shown that factor analysis is useful for evaluating the construct structure of an instrument, including assessing item validity and identifying the dominant dimensions of mathematical abilities (Sürücü et al., 2022; Rogers, 2022). In the present study, the factor structure indicated that generalization and representation were integrated within one dominant factor, whereas procedural ability, represented by transformation, and conceptual meaning emerged as distinct dimensions (Kieran, 2022; Raviv et al., 2022). Within the specific instructional context examined, the area model supported by PhET Interactive Simulation functioned not only as a visualization tool but also as a context in which these dimensions of algebraic thinking could be observed (de Sousa & Alves, 2022; Artasari et al., 2024).

The prominence of the visual-representational factor suggests that students' responses were strongly organized around connections between visual and symbolic forms (Ünal et al., 2023; Wilkie, 2024). This result is consistent with studies describing the role of digital technology in supporting connections among representations and conceptual understanding (Laurence, 2022; Akinoso et al., 2025). The separate procedural-transformation factor indicates that algebraic manipulation remained an important dimension but was empirically differentiated from visual representation and conceptual meaning (Pelayo et al., 2023; Rafiepour et al., 2023). The emergence of conceptual meaning as a distinct factor further suggests that interpreting the mathematical meaning of algebraic expressions may involve processes that are not fully captured by procedural or representational performance alone (Harel, 2025; Alam & Mohanty, 2024). Overall, the factor structure supports the multidimensional nature of algebraic thinking (Kieran, 2022; Sun et al., 2023). Factor analysis therefore served not only as a data-reduction technique but also as an analytical approach for examining the dimensions of mathematical ability observed in a technology-supported learning context (Hair et al., 2020; Griffith, 2024).

These findings have implications for mathematics teaching, particularly for the development of algebraic thinking. The factor integrating generalization and visual representation indicates that activities involving the area model and PhET may help students connect visual and symbolic representations before engaging in formal algebraic manipulation. Teachers may therefore use visual-interactive representations during the initial stages of instruction to support pattern generalization and translation between visual and symbolic forms. However, because conceptual meaning emerged as a relatively separate factor, conceptual understanding should not be assumed to develop automatically from procedural activities. Instruction should explicitly require students to explain their mathematical reasoning, relate each component of a visual model to its algebraic meaning, and reflect on the relationship between the area model and the corresponding symbolic expression. The findings may therefore inform the design of algebra instruction that balances visual representation, procedural fluency, and conceptual understanding.

Conclusion

This study identified the factor structure of students' algebraic thinking within an instructional context involving an area model supported by PhET Interactive Simulation. From the initial 16 indicators, four indicators were removed based on inadequate MSA values, resulting in 12 indicators suitable for further analysis. The factor analysis produced four components with eigenvalues greater than 1, explaining 62.06% of the total variance before rotation.

The rotated solution revealed three interpretable dimensions of students' algebraic thinking. The first was a visual-representational factor that integrated generalization and representation, with R4 showing the highest loading (.781). The second was a procedural-transformational factor characterized by algebraic manipulation, with T1 showing the highest loading (.768). The third was a conceptual-meaning factor represented by C1–C4, with C3 showing the highest loading (.756). The fourth component had no clearly dominant group of indicators and was therefore treated as a residual or weak component rather than a clearly established dimension.

These findings indicate that students' algebraic thinking in the examined learning context is multidimensional. Visual-representational ability, procedural transformation, and conceptual meaning showed distinguishable patterns, suggesting that algebra instruction should integrate visual representation, symbolic manipulation, and explicit conceptual explanation. The findings should nevertheless be interpreted as exploratory and context-specific because the analysis was conducted with 128 students participating in an area model and PhET-supported learning context. Future studies should use larger and independent samples and apply Confirmatory Factor Analysis to test the stability and fit of the identified structure.

Acknowledgments
The authors express their gratitude to the Directorate of Learning and Student Affairs (*Direktorat Pembelajaran dan Kemahasiswaan*, Belmawa), Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia, for funding this study through the 2026 Student Creativity Program–Scientific Article scheme (*Program Kreativitas Mahasiswa–Artikel Ilmiah*, PKM-AI).

References

Akinoso, S. O., Olaniyi, O. A., & Akinoso, H. O. (2025). Effect of Phet Simulations on Secondary School Students' Achievement in Elementary Algebra. *Lagos Education Review*, *20*(1), 506-516. https://doi.org/10.52968/15062424

Alam, A., & Mohanty, A. (2024). Unveiling the complexities of 'Abstract Algebra'in University Mathematics Education (UME): fostering 'Conceptualization and Understanding'through advanced pedagogical approaches. *Cogent Education*, *11*(1), 2355400. https://doi.org/10.1080/2331186X.2024.2355400

Álvaréz, P. A. B., Chacón-Castro, M., Busain, R. S., & Jadán-Guerrero, J. (2024). A didactic proposal for teaching factorization cases of expressions of the form ax2+ by+ cx+ ay+ exy+ f through Mathigon. *Eurasia Journal of Mathematics, Science and Technology Education*, *20*(10), em2514. https://doi.org/10.29333/ejmste/15198

Artasari, A., & Oktaviyanthi, R. (2024). Development Strategy of E-Pocket Book to Optimize Algebraic Thinking Skills of Junior High School Students. *Jurnal Eduscience*, *11*(3), 568-587. https://doi.org/10.36987/jes.v11i3.6425

Artasari, A., Oktaviyanthi, R., & Delvianingsih, N. (2024). Optimizing Algebraic Thinking Using the Area Model Algebra Worksheet Based on Phet Interactive Simulation. *Plusminus: Jurnal Pendidikan Matematika: Institut Pendidikan Indonesia*, *4*(2), 241-254. https://doi.org/10.21831/jitp.v11i4.79362

Brandenburg, N., & Papenberg, M. (2024). Reassessment of innovative methods to determine the number of factors: A simulation-based comparison of exploratory graph analysis and next eigenvalue sufficiency test. *Psychological Methods*, *29*(1), 21. https://psycnet.apa.org/doi/10.1037/met0000527

de Sousa, R. T., & Alves, F. R. V. (2022). Quadratic functions and phet: An investigation from the perspective of the theory of figural concepts. *Contemporary Mathematics and Science Education*, *3*(1), ep22010. https://doi.org/10.30935/conmaths/11929

Griffith, J. W. (2024). Exploratory factor analysis. In *Encyclopedia of quality of life and well-being research* (pp. 2313-2315). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-17299-1_969

Hair Jr, J. F., Howard, M. C., & Nitzl, C. (2020). Assessing measurement model quality in PLS-SEM using confirmatory composite analysis. *Journal of business research*, *109*, 101-110. https://doi.org/10.1016/j.jbusres.2019.11.069

Harel, G. (2025). Promoting linear algebraic reasoning among students: Affordances and challenges. *PRIMUS*, *35*(9-10), 930-950. https://doi.org/10.1080/10511970.2024.2327325

Hayes, C., & Proulx, M. J. (2024). Turning a blind eye? Removing barriers to science and mathematics education for students with visual impairments. *British Journal of Visual Impairment*, *42*(2), 544-556. https://doi.org/10.1177/02646196221149561

Kabar, M. G. D. (2023). A thematic review of quadratic equation studies in the field of mathematics education. *Participatory Educational Research*, *10*(4), 29-48. https://doi.org/10.17275/per.23.58.10.4

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: background, overarching dimensions, and new directions. *ZDM–Mathematics Education*, *54*(6), 1131-1150. https://doi.org/10.1007/s11858-022-01435-6

Kriegler, S. (1999) 'Just what is algebraic thinking?', *CMC ComMuniCator*, 23(3), pp. 32–36.

Laurence C, L. (2022). Integration of PhET interactive simulations in online synchronous and asynchronous teaching of science: It's impact on learners' science process skills. *International Journal of Trend in Scientific Research and Development*, *6*(6), 61-77. http://www.ijtsrd.com/papers/ijtsrd51801.pdf

Lee, M. Y., & Yeo, S. (2026). Exploring preservice teachers' problem-solving with fractopia: A dynamic digital tool developed to support semiotic mediation in fraction learning. *Journal of Mathematics Teacher Education*, 1-39. https://doi.org/10.1007/s10857-026-09746-1

Lowrie, T., & Logan, T. (2023). Spatial visualization supports students' math: Mechanisms for spatial transfer. *Journal of Intelligence*, *11*(6), 127. https://doi.org/10.3390/jintelligence11060127

Nájera, P., Abad, F. J., & Sorrel, M. A. (2025). Is exploratory factor analysis always to be preferred? A systematic comparison of factor analytic techniques throughout the confirmatory–exploratory continuum. *Psychological Methods*, *30*(1), 16. https://psycnet.apa.org/doi/10.1037/met0000579

Ngoveni, M. A. (2025). Deconstructing and addressing factorizing errors and misconceptions in a TVET college: Mathematical insights and interventions. *The International Journal of Science, Mathematics and Technology Learning*, *32*(2), 23-47. https://doi.org/10.18848/2327-7971/CGP/v32i02/23-47

Osborne, J. W., & Kellow, J. T. (2008). Exploratory Factor Analysis. *Best practices in quantitative methods.*

Pelayo, E. O., Lomibao, L. S., & Tan, R. G. (2023). Conceptual understanding vs. procedural fluency: A literature review on the mathematics teachers emphasis in teaching of rational algebraic expressions. *Journal of Innovations in Teaching and Learning*, *3*(1), 54-59. https://doi.org/10.12691/jitl-3-1-10

Perkins, K. (2020). Transforming STEM learning at scale: PhET interactive simulations. *Childhood Education*, *96*(4), 42-49. https://doi.org/10.1080/00094056.2020.1796451

Prain, V., & Tytler, R. (2022). Theorising learning in science through integrating multimodal representations. *Research in Science Education*, *52*(3), 805-817. https://doi.org/10.1007/s11165-021-10025-7

Rafiepour, A., Faramarzpour, N., & Fadaee, M. R. (2023). Introducing a Teaching Technique for Reducing Students' Mistakes in Simplifying Algebraic Expressions. *Mathematics Teaching Research Journal*, *15*(5), 193-208. https://files.eric.ed.gov/fulltext/EJ1412246.pdf

Raviv, L., Lupyan, G., & Green, S. C. (2022). How variability shapes learning and generalization. *Trends in cognitive sciences*, *26*(6), 462-483. https://doi.org/10.1016/j.tics.2022.03.007

Rogers, P. (2022). Best practices for your exploratory factor analysis: A factor tutorial. *Revista de Administração Contemporânea*, *26*(06), e210085. https://doi.org/10.1590/1982-7849rac2022210085.en

Sun, S., Sun, D., & Xu, T. (2023). The developmental progression of early algebraic thinking of elementary school students. *Journal of Intelligence*, *11*(12), 222. https://doi.org/10.3390/jintelligence11120222

Sürücü, L., Yıkılmaz, İ., & Maşlakçı, A. (2022). Exploratory factor analysis (EFA) in quantitative researches and practical considerations. *Gümüşhane Üniversitesi Sağlık Bilimleri Dergisi*, *13*(2), 947-965. https://doi.org/10.37989/gumussagbil.1183271

Ünal, Z. E., Ala, A. M., Kartal, G., Özel, S., & Geary, D. C. (2023). Visual and Symbolic Representations as Components of Algebraic Reasoning. *Journal of Numerical Cognition*, *9*(2), 327-345. https://doi.org/10.5964/jnc.11151

Vale, I., & Barbosa, A. (2023). Visualization: A pathway to mathematical challenging tasks. In *Mathematical challenges for all* (pp. 283-306). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-031-18868-8_15

Wahab, A. (2026). Translating Cyclic Mathematical Representations: Effects on Students' Learning Outcomes and Cognitive Processes in Covariation Problem-Solving. *Journal of Teaching and Learning*, *20*(2). https://doi.org/10.22329/jtl.v20i1.9808

Wilkie, K. J. (2024). Coordinating visual and algebraic reasoning with quadratic functions. *Mathematics Education Research Journal*, *36*(1), 33-69. https://doi.org/10.1007/s13394-022-00426-w

Yao, X. (2022). Representation transformations in transition from empirical to structural generalization. *The Journal of Mathematical Behavior*, *66*, 100964. https://doi.org/10.1016/j.jmathb.2022.100964